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**SOLVING METHOD FOR FUZZY MULTIPLE OBJECTIVE  
INTEGER OPTIMIZATION**

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***Abstract***

*Starting from the idea of Wang and Liao (2001) for solving fuzzy non-linear integer programming problem and taking into account the multiple criteria optimization in fuzzy environment, a solving method for fuzzy multiple objective integer optimization problem is developed here. Theoretical analysis on efficient solutions for multiple criteria optimization problem and computational results are also presented.*

***Keywords:*** *fuzzy programming, integer programming, multiple criteria*

**1. INTRODUCTION**

Starting from the idea of Wang and Liao [7] for solving fuzzy non-linear integer programming problem and taking into account the multiple criteria optimization in fuzzy environment, a solving method for fuzzy multiple objective integer optimization problem is developed here. Wang and Horng [8] proposed an approach to perform complete parametric analysis in integer programming by considering all of the possible candidates of fuzzy environment parameters in constraints. We use their methods to solving multiple objective programming with integer variables and fuzzy constraints.

In [2], Perkgoz et al. focused on multiple objective integer programming problems with random variable coefficients in objective functions and/or constraints. In [5], Sakawa and Kato presented an interactive fuzzy satisfying method for nonlinear integer programming problems through a genetic algorithm. In [7]  $\alpha$ -Pareto optimal solutions were determined for the multiple objective integer nonlinear programming problems having fuzzy parameters in the constraints together with the corresponding stability set of the first kind. Cite [1] presented a method useful in solving a special class of large-scale multiple objective integer problems based upon a combination of the

decomposition algorithm coupled with the weighting method together with the branch-and-bound method.

Section 2 describe the mathematical model of a fuzzy multiple objective integer optimization. A solving method for such a model is presented in Section 2 using an algorithm which works on deterministic problems and a concept of principal candidates for fuzzy environment parameters. Computational results are inserted in Section 4. The considered example solved a multiple criteria linear fractional programming problem having fuzzy constraints and integer decision variables. Section 5 is for short concluding remarks.

## 2. FUZZY MULTIPLE OBJECTIVE INTEGER OPTIMIZATION MODEL

Consider the multiple objective programming with fuzzy constraints and integer variables

$$\text{"min"} \{z(x) = (z_1(x), z_2(x), \dots, z_p(x) \mid x \in \bar{X})\}, \quad (2.1)$$

- $\bar{X} = \{x \in R^n \mid Ax \leq b, x \geq 0, x \in Z\}$  is the feasible set of the problem,
- $A$  is an  $m \times n$  constraint matrix,  $x$  is an  $n$ -dimensional vector of decision variable and  $b \in R^m$ ,
- $p \geq 2$ ,
- Vector  $(z_i(x))_{i=1,p}$  represents the objective functions which could be linear, linear fractional or convex functions (in order to make the new method workable).

The term "min" used in Problem (2.1) is for finding all efficient solutions in a minimization sense in terms of the Pareto optimality.

A possible way to handle constraints imprecision is to consider the following parametric problem

$$\text{"min"} \{z(x) = (z_1(x), z_2(x), \dots, z_p(x) \mid x \in X(\theta))\}. \quad (2.2)$$

We noted  $X(\theta) = \{x \in R^n \mid Ax \leq b + \theta b', x \geq 0, x \in Z\}$  the feasible set of the problem,  $\theta \in [0,1]$  and  $b'$  a given perturbation vector ([7]).

## 3. THE SOLVING METHOD

In [3] an algorithm to solving multiple objective linear fractional programming (MOLFP) is presented. We insert below a briefly description of this algorithm.

Step 1 Establish the problem constant and put  $i = 0$ .

Step 2 Compute marginal values using ERA ([3]).

Step 3 Choose  $x^0$  as a feasible solution of MOLFP.

Step 4 If  $\nabla z(x^i) = 0$  then  $x^i$  is the optimal solution. Otherwise go to Step 5.

Step 5 (Acceptability test) If  $x^i$  is an acceptable solution of MOLFP then favorable

STOP If  $x^i$  is not an acceptable solution of MOLFP but a possible improving exists go to Step 6

Step 6 Otherwise unfavorable STOP. Convenient solution does not exist for MOLFP.

Step 6 Search a direction of minimization  $s$  for as many objective functions as possible.

If there is no such a direction  $s$  then let  $s$  be the direction of minimization for the criterion

which realized the largest slack in Step 5. Compute  $\lambda_{\max}$  and according with ERA go to Step 7 with  $x^{i+1} = x^i + \lambda_{\max} s$ .

Step 7 If  $S(x^{i+1}) \leq S(x^i)$  then go to Step 4 with  $i = i + 1$ . Otherwise, compute  $x^{i+1} = x^i + \lambda s$  for  $\lambda < \lambda_{\max}$  and return to the test of Step 7.

We will use the above algorithm (under the name *MultiObj*( $\theta$ )) to solve Problem (2.2) meaning the deterministic problem with the feasible set  $X(\theta)$ . To improve the interactivity of the method different values for  $\theta$  will be considered. Wang and Horng [8] proposed an approach to perform complete parametric analysis in integer programming by considering all of the possible candidates of  $\theta$ . They defined the principal candidates of  $\theta$  as been that  $\theta$  which makes  $b_i + \theta b'_i$  an integer for  $i = \overline{1, m}$ . We also work with these  $\theta$ 's. In [7], Wang and Liao proposed a heuristic algorithm to analyze the same fuzzy problem. We will use their method to solving multiple objective programming with integer variables and fuzzy constraints.

For a fixed value  $\theta$  we start defining Problem (3.1) as Problem (2.2) without the integrity restriction of variables.

$$\text{"min"} \{z(x) = (z_1(x), z_2(x), \dots, z_p(x)) \mid Ax \leq b + \theta b', x \geq 0\} \quad (3.1)$$

We obtain an efficient solution for Problem (3.1) using *MultiObj*(1). It is a solution for Problem (2.2) if and only if its components are integer numbers. In this case it is also solution for Problem (2.1) but with minimal degree in fuzzy environment. Consequently, our next goals are to transform the solution into an integer one and also to improve its fuzzy degree. These goals could be attend modifying values  $x_i$  in  $[x_i]$  or  $[x_i] + 1$  such that the deviation of the perturbation vector decreases using *back*( $p, r, y$ ) procedure described below.

Procedure *back*( $p, r, y$ ):

- If  $p = n$  then if  $Ax \leq b + \theta_r b'$  then  $y = (x^r, \theta_r, z(x^r))$ ; return.
- If  $u[1] = 0$  then  $x[p] = [x[p]]$ ;  $u[1] = 1$ ; *back*( $p + 1, r, y$ );  $u[1] = 0$ .
- If  $u[2] = 0$  then  $x[p] = [x[p]] + 1$ ;  $u[2] = 1$ ; *back*( $p + 1, r, y$ );  $u[2] = 0$ .

Taking into account the above remarks the solving algorithm could be described as follows.

Step 1 Define the thresholds  $0 = \theta_1 < \theta_2 < \dots < \theta_q = 1$  using the principal candidates of parameters  $\theta$ . Put  $q = 1$ .

Step 2 For  $k = q$  down to 1 do

- Compute values  $x^k = (x_1, x_2, \dots, x_n)$  using *MultiObj*( $\theta_k$ ).

- If  $x^k \in Z^n$  then favorable STOP with  $x^k$  the  $\theta_k$  – fuzzy degree acceptable solution of the problem.
- Otherwise call  $back(1, k, y^k)$  and obtain  $x^k = (x'_1, x'_2, \dots, x'_n)$ . Identify  $k_{\min}$  such that  $Ax \leq b + \theta_{k_{\min}} b'$ . Then  $x^{k_{\min}}$  is the  $\theta_{k_{\min}}$  – fuzzy degree acceptable solution of the problem.

Step 3 Describe the problem solution as  $(y^k)_{k=1, \dots, q}$ .

#### 4. COMPUTATIONAL RESULTS

In order to illustrate our solving method let us consider the following deterministic linear fractional program

$$\text{"min"} \left( z(x) = \left( \frac{2x_1 - 7x_2 + 10}{-x_1 - 2x_2 + 50}, \frac{-x_1 - x_2 + 17}{x_1 - x_2 + 13} \right) \right) \quad (4.1)$$

Subject to

$$\begin{cases} -x_1 + 3x_2 \leq 6, \\ 3x_1 + x_2 \leq 12, \\ x_1, x_2 \geq 0, \\ x_1, x_2 \in Z. \end{cases} \quad (4.2)$$

The fuzzy feasible set of Problem (4.1) meaning (4.2) is treated as a parameter feasible set (4.3) considering  $\theta \in [0, 1]$  and

$$\begin{cases} -x_1 + 3x_2 \leq 6 + 3\theta, \\ 3x_1 + x_2 \leq 12 + 6\theta, \\ x_1, x_2 \geq 0, \\ x_1, x_2 \in Z. \end{cases} \quad (4.3)$$

Step 1 gave  $q=7$  and  $\theta = (0, 0.166, 0.334, 0.5, 0.666, 0.834, 1)$ . The solving algorithm gave fuzzy integer solutions contained in Table 4.1. Consequently, a solution of Problem (4.1) is

$$Y = [y^1 = ((3, 3), 0, (-0.21, 0.366)), y^2 = ((4, 4), 0.834, (-0.26, 0.3))]$$

$k$	$\theta$	$x_1$	$x_2$	$f(x_1)$	$f(x_2)$	$x_1' \in Z$	$x_2' \in Z$
1	0	3	3	-0.121	0.366	3	3
2	0.166	3.249	3.249	-0.155	0.35	3	3
3	0.334	3.501	3.501	-0.19	0.33	3	3
4	0.5	3.75	3.75	-0.225	0.316	3	3

5	0.666	3.9	3.9	-0.24	0.3	3	3
6	0.834	4.251	4.251	-0.3	0.28	4	4
7	1	4.5	4.5	-0.34	0.26	4	4

Table 4.1

To deal with linear fractional optimization we used classic solving methods which are described in detail in [6]. Also, to make basic optimization calculus we used classic tools. To perform global computational analyze we implement our algorithms (ERA,  $MultiObj(\theta)$ ,  $back(p, r, y)$ ).

## 5. CONCLUDING REMARKS

In this paper we have proposed a method of solving fuzzy (constraints) multiple objective integer (decision variables) optimization problems.

We worked with the concept of principal candidates for the fuzzy environment parameters (and to improve the interactivity of the method different values for parameters  $\theta$  were considered) and obtained a fuzzy solution meaning possible solution values versus fuzzy degree of this. We have applied an algorithm which solves deterministic classic problems and then we transformed the solution into an integer one (developing procedure  $back(p, r, y)$ ) also improving its fuzzy degree.

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